Effect of Stress – Strain Relationship on the Elasticity Modulus and Moment Capacity

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Abstract:

The Effect of strain on the elasticity modulus and moment capacity of members are studied using different types of stress – strain relationships (linear, elasto-plastic, elastic perfectly plastic, bi-linear, tri-linear and other continuous models) for bending with or without axial loads using the steps of solution shown in the research.

Generally in all cases inelastic modulus reduced with increasing the strain and stress level and significant decrease is occurred in the value of inelastic modulus, the value of inelastic modulus reduces to about (50%) of its initial value when the stress value within (0.4-0.5) of the yield strength. Effect of stress ratio (R_F) and strain ratio ($R\varepsilon$) are also studied. Suitable equations are proposed to estimate the values of inelastic modulus and moment capacity.

تأثيرعلاقة الأجهاد – الأنفعال على معامل المرونـة ومقاومة العزوم

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الخلاصة:

تم في هذا البحث دراسة تأثير الأنفعال على معامل المرونة ومقاومة العزوم باستخدام أنواع مختلفة من علاقات الاجهاد-الانفعال (linear, elasto-plastic, elastic perfectly plastic, bi-linear, tri-linear and other continuous models) لحالة الانثناء بدون أو مع وجود الاحمال المحورية بأستخدام الخطوات وأتباع مخطط الحل المبين في متن البحث.

بصورة عامة في جميع الحالات فأن قيمة معامل المرونة تقل بزيادة قيمة الانفعال والاجهاد ويحصل هبوط كبير في قيمة معامل المرونة عندما يكون قيمة الاجهاد يتراوح بين (0.4-0.5) من قيمة أجهاد الخضوع حيث يفقد (50%) من قيمتها الاولية. كما تم دراسة تأثير نسبة الأجهاد (R_F) و نسبة الأنفعال ($R\varepsilon$). وتم أقتراح معادلات مناسبة لحساب قيمة معامل المرونة ومقاومة العزوم.

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Notation:

A = Area of the section.
b = width of the section.
E = Initial elastic modulus.
E' = inelastic modulus in bending only.
E" = inelastic modulus in bending with axial load.
h = depth of the section.
h1 & h2 = distances from neutral axis to the extreme fiber.
I = Moment of inertia of the section.
M = Moment capacity of the section.
y = distance from the neutral axis to the specified point.
$\beta_1 = E'/E$ (modulii ratio in bending case only).
$\beta_2 = E''/E$ (modulii ratio in bending with axial loading).
$\Delta = \varepsilon_1 - \varepsilon_2$ (algebraic difference between the two extreme strain).
$\varepsilon_o = \sigma_o / E$ (applied strain corresponding to the stress).
ε_y = yield strain corresponding to yield strength = σ_y / E.
ρ = radius of curvature.
σ = Stress at strain (ε).
$\psi = \beta \Delta$ (moment capacity coefficient).

Introduction:

Plastic limit analysis which is commonly used for the analysis of steel structures is not completely applicable for the analysis of reinforced concrete structures. For steel structures in flexure, a collapse load analysis based on the assumption of elastic-perfectly plastic bending moment-curvature behavior provides the basis for the plastic limit analysis. The extensive ductility of physical steel sections at plastic moment provides realism to the basic assumption and the existence of a strain-hardening range provides a factor of safety. Reinforced concrete beams can sometimes exhibit a softening response in which the load, after reaching its peak value, does not follow a constant load yield plateau but gradually declines with increasing displacement because of the limited ductility and strain-softening behavior of typical reinforced concrete sections.

This paper is a trial to study the inelastic behavior of members in bending with and without axial strain for different types of stress-strain relationships.

The basis of plastic analysis is that the structural material used should be ductile [1], initially for idealized stress-strain relationship shown in Fig. (1), the material behaves elastically but after the yield point has been reached it continues to deform at a constant stress level known as the yield strength (Fy), this phenomenon of yielding at constant stress is known as plastic yielding. Due to absence of yield plateau, the applications of plastic limit analysis to reinforced concrete beams are not applicable.

The basic assumptions of bending [2 – 4] when the material is stressed less than the proportional limit are that the section of the member remains plane during bending, and hence longitudinal strains are proportional to their distances from the neutral axis. The center plane is a neutral surface, hence all strains are zero at that plane, and the depth of the member in not compressible, and hence the strain of the section in the transverse direction along the depth is neglected. Fig. (2) Shows the bending of the member of rectangular cross section, the unit elongation of a fiber at distance (y) from the neutral surface is:

$$\varepsilon = y / \rho \qquad \text{-------- (1)}$$

Where (ρ) is the radius of curvature of the neutral surface produced by the bending moment. At lower and upper surface of the member, the elongation become:-

$$\varepsilon_1 = h_1 / \rho \qquad \text{--------- (2)}$$

$$\varepsilon_2 = h_2 / \rho \qquad \text{--------- (3)}$$

From static:

$$\int \sigma \, dA = b \int \sigma \, dy = 0 \qquad \text{--------- (4)}$$

$$\int \sigma y \, dA = b \int \sigma \, y \, dy = M \qquad \text{-------- (5)}$$

The above two equations represent the sum of normal forces and moment acting on any cross section with respect to neutral axis.

From equation (1); $y = \rho \, \varepsilon$

Differentiate both sides to get:

$dy = \rho \, d\varepsilon$

Substituting dy in equation (4) to get:

$b \int \sigma \, y = b \int \sigma \, \rho \, d\varepsilon = 0$ -------- (6)

Let (Δ) be the algebraic difference between the two extreme strains (ε_1 & ε_2).

$\Delta = \varepsilon_1 - \varepsilon_2 = h_1/\rho - (-h_2/\rho) = (h_1+h_2)/\rho = h/\rho$ -------- (7)

Or $\rho = h/\Delta$

Now consider the general stress-strain relationship shown in Fig. (3-a).
Values of (ε_1 & ε_2) are proportional to the distances from the neutral axes, substituting (y and dy) into equation (5) to get:

$b \, \rho^2 \int \sigma \, \varepsilon \, d\varepsilon = M$ ------- (8)

Substituting ($\rho = h/\Delta$) into equation (8), the following equation is obtained:-

$(12 \, I / \rho \, \Delta^3) \int \sigma \, \varepsilon \, d\varepsilon = M$ ------- (9)

Let $E' = (12/\Delta^3) \int \sigma \, \varepsilon \, d\varepsilon$ ------- (10)

Equation (9) can be written in another form:

$E' \, I / \rho = M = E' \, I \, \Delta / h$ ------- (11)

This form is similar to the equation of simple bending and (E') represent the inelastic modulus in case of pure bending.

When the tension and compression portions of the stress-strain diagram are the same and in bending case only without axial load, the neutral axis passes through the centeroid of the rectangular section, hence:

$h_1 = h_2 = h/2$ and $\varepsilon_1 = - \varepsilon_2 = \Delta/2$ and then equation (10) become:

$E' = (24/\Delta^3) \int \sigma \, \varepsilon \, d\varepsilon$

Steps of solution:-

1- For a given value of strain (Δ), do the following steps.
2- From the assumed stress-strain relationship, estimate the value of (ε_1 & ε_2) in such a manner to keep the value of the stress constant.

3- Find the value of the integral ($Ig = \int \sigma \varepsilon \, d\varepsilon$).

4- Find the value of the inelastic modulus ($E' = 12 \, Ig / \Delta^3$).
5- Find the corresponding value of moment capacity ($M = E' I / \rho$).
6- Repeat steps 1 to 5 for other values of (Δ).
7- Plot the curve of (M and E') versus the strain (Δ).
8- Repeat all the above steps for different types of stress-strain relationship (linear, elasto-plastic, elastic perfectly plastic, bi-linear, tri-linear and other continuous models).

For members subjected to bending and compression force together, such as produced by eccentrically applied compressive force. The position of neutral axis is determined by the values of (ε_1 & ε_2) as is shifted from its position for pure bending by amount (ε_o) which is caused by the applied load (P). Strain at any point can be determined by adding (y/ρ) to the applied compressive strain (ε_o) as follow:

$$\varepsilon = \varepsilon_o + y / \rho$$

$$y = \rho (\varepsilon - \varepsilon_o) \qquad \text{and } dy = \rho \, d\varepsilon$$

From static, sum of normal stresses equal to the applied force (P):

$$P = - b \int \sigma \, dy = - b \rho \int \sigma \, d\varepsilon = -b \, h/\Delta \int \sigma \, d\varepsilon \qquad \text{-------- (12)}$$

$$\sigma_c = P/b \, h = -1/\Delta \int \sigma \, d\varepsilon \qquad \text{------- (13)}$$

By same way, sum of moment with respect to the neutral axis is given by the expression:

$$M = b \int \sigma \, y \, dy = b \rho^2 \int \sigma (\varepsilon - \varepsilon_o) d\varepsilon \qquad \text{------ (14)}$$

Using the same simple transformation, the above equation become:

$$M = (12 \, I / \rho \, \Delta^3) \int \sigma (\varepsilon - \varepsilon_o) d\varepsilon \qquad \text{------- (15)}$$

Finally $M = E'' I / \rho = E'' I \Delta/h$ ------- (16)

Where $E''= 12/\Delta^3 \int \sigma (\varepsilon - \varepsilon_o)d\varepsilon$ ------- (17)

Equations (16 and 11) are similar to the equation of bending moment in elementary strength of material, and (E”) represent the inelastic modulus in case of combined bending and compressive load.

Steps of solution:-

1- For a given value of (P), find $\sigma_c = P/A$.
2- For a given value of strain (Δ), do the following steps.
3- From the assumed stress-strain relationship, find the value of (ε_o) which corresponds to (σ_c).
4- From the assumed stress-strain relationship, estimate the value of (ε_1 & ε_2) in such a manner to keep the stress (σ_c) constant.
5- Find the value of the integral [$Ig = \int \sigma (\varepsilon - \varepsilon_o) d\varepsilon$].
6- Find the value of the inelastic modulus ($E'' = 12 Ig /\Delta^3$).
7- Find the corresponding value of moment capacity ($M= E'' I /\rho$).
8- Repeat steps 2 to 7 for other values of (Δ).
9- Repeat steps 1 to 8 for other values of (σ_c).
10- Plot the curve of (M and E”) versus the strain (Δ) for different values of (σ_c).
11- Repeat all the above steps for different types of stress-strain relationship as before.

Analysis and Results:

Inelastic behavior of the following stress-strain relationships (linear, elasto-plastic, elastic perfectly plastic, bi-linear, tri-linear and other continuous models) are studied, inelastic modulus in pure bending and with axial strain are derived using the previous method of solution. For the analysis assume ($\beta_1= E'/E$ and $\beta_2= E''/E$).

A- Linear stress-strain relationship shown in Fig. (3-b):

$\beta_1 = 1$ and $E'=E$

$M = E\,I\,\Delta/\,h$

$\beta_2 = [4(\varepsilon_1^{3} - \varepsilon_2^{3}) - 6\,\varepsilon_o\,(\varepsilon_1^{2} - \varepsilon_2^{2})\,]\,/\,\Delta^3$

B- Elasto - Plastic stress-strain relationship shown in Fig. (3-c):

$\beta_1 = 1$ for $\varepsilon \le \varepsilon_y$

$\beta_1 = 3\,(\varepsilon_y/\Delta) - 4\,(\varepsilon_y/\Delta)^3$ for $\varepsilon > \varepsilon_y$

$M = E\,I/h\ \ \beta_1\,\Delta$

Let $\psi = \beta_1\,\Delta = M\,h\,/\,E\,I$

Steps to determine (β_2):

1- For a given (σ_c), find ($\varepsilon_o = \sigma_c\,/\,E$).

2- Take a value for (ε_1):

If $\varepsilon_1 \le \varepsilon_{y:}$ take a value for (ε_2):

2.1- If $\varepsilon_2 \le \varepsilon_{y:}$; $\varepsilon_2 = -2\,\varepsilon_o - \varepsilon_1$

2.2- If $\varepsilon_2 > \varepsilon_{y:}$ $\varepsilon_2 = (\varepsilon_1^{2}/2 - 3/2\,\varepsilon_y^{2} + \varepsilon_o\,\varepsilon_y)/(\varepsilon_o + \varepsilon_y)$.

3- If $\varepsilon_1 > \varepsilon_{y:}$ take a value for (ε_2):

3.1- If $\varepsilon_2 \le \varepsilon_{y:}$ $\varepsilon_2 = -\,\varepsilon_o + \sqrt{\,}\,\varepsilon_o^{2} + 2(\varepsilon_1\,\varepsilon_y\ -\varepsilon_y^{2}/2 + \varepsilon_o\,\varepsilon_1)$.

3.2- If $\varepsilon_2 > \varepsilon_y$; $\varepsilon_2 = (\varepsilon_1\,\varepsilon_y\ - 2\,\varepsilon_y^{2} + \varepsilon_o\,\varepsilon_1)/(\varepsilon_o + \varepsilon_y)$.

4- Find $\Delta = (\varepsilon_1 - \varepsilon_2)$.

5- Find (β_2):

5.1- If $\varepsilon_1 \le \varepsilon_{y:}$ $\beta_2 = [4(\varepsilon_1^{3} - \varepsilon_2^{3}) - 6\,\varepsilon_o\,(\varepsilon_1^{2} - \varepsilon_2^{2})\,]\,/\,\Delta^3$

5.2- If $\varepsilon_1 > \varepsilon_{y:}$ $\beta_2 = 12/\Delta^3\,[(\varepsilon_y^{3} - \varepsilon_2^{3})/3 - \varepsilon_o/2\,(\varepsilon_y^{2} - \varepsilon_2^{2}) + \varepsilon_y\,\{(\varepsilon_1^{2} - \varepsilon_2^{2}) - \varepsilon_o\,(\varepsilon_1 - \varepsilon_y)\}]$

6- Find $E'' = \beta_2\,E$ and $M = E\,I/h\ \ \beta_2\,\Delta$.

7- Repeat the steps 2-6 for other values of (ε_1).

8- Repeat the steps 1-7 for other values of (σ_c).

9- Plot the curve of M and E"/E versus (Δ).

C- Elastic – Perfectly plastic stress-strain relationship shown in Fig. (3-d):

$\beta_1 = 3\,\varepsilon_y/\Delta$

$E' = 3\,Fy/\Delta$

$M = EI/h\,(3\varepsilon_y) = 3\,Fy\,I/h$

$\beta_2 = 12\,\varepsilon_y/\Delta^2\,[(\varepsilon_1 + \varepsilon_2)\,/2 - \varepsilon_o]$

D- Parabolic concrete stress-strain relationship [5] shown in Fig. (3-e):

$\sigma = K\,fc'\,[\,2\,(\varepsilon/\varepsilon_o) - (\varepsilon/\varepsilon_o)^2\,]$ for $\varepsilon \le \varepsilon_o$

$\sigma = K\,fc'$ for $\varepsilon > \varepsilon_o$

$E' = 500\,K\,fc'\,[1 - 187.5\,\Delta]$ for $\varepsilon \le \varepsilon_o$

$E' = ¾\,K\,fc'/\,\Delta$ for $\varepsilon > \varepsilon_o$

$E'' = 500\,K\,fc'\,[1+250\,\varepsilon_o - 187.5\,\Delta - 1.5\,\varepsilon_o/\Delta]$ for $\varepsilon \le \varepsilon_o$

$E'' = ¾\,K\,fc'/\Delta\,(1-2\,\varepsilon_o/\Delta)$ for $\varepsilon > \varepsilon_o$

E- Bi-linear stress-strain relationship [6] shown in Fig. (4-a):

$\beta_1 = 1$ for $\varepsilon \le \varepsilon_{y1}$

$\beta_1 = E2/E1 + 3(1-E2/E1)\, \varepsilon_{y1}/\Delta - 4(1-E2/E1)\, (\varepsilon_{y1}/\Delta)^3$ for $\varepsilon_{y2} \geq \varepsilon > \varepsilon_{y1}$

$\beta_2 = [4(\varepsilon_1^3 - \varepsilon_2^3) - 6\, \varepsilon_o\, (\varepsilon_1^2 - \varepsilon_2^2)\,] / \Delta^3$ for $\varepsilon \leq \varepsilon_{y1}$

$\beta_2 = E2/E1\, (1-3\, \varepsilon_o/\Delta) + 3(1-E2/E1)\, (1-4\varepsilon_o/\Delta)(\varepsilon_{y1}/\Delta) + 12(1-E2/E1)\, (\varepsilon_o/\Delta)\, (\varepsilon_{y1}/\Delta)^2 - 4(1-E2/E1)\, (\varepsilon_{y1}/\Delta)^3$ for $\varepsilon_{y2} \geq \varepsilon > \varepsilon_{y1}$

These equations give the same solution of linear relationship when (E2=E1) and same as elasto-plastic relationship when (E2=0).

F- Tri-linear stress-strain relationship [7] shown in Fig. (4-b):

$\beta_1 = 1$ for $\varepsilon \leq \varepsilon_{y1}$

$\beta_1 = E2/E1 + 3(1-E2/E1)\, \varepsilon_{y1}/\Delta - 4(1-E2/E1)\, (\varepsilon_{y1}/\Delta)^3$ for $\varepsilon_{y2} \geq \varepsilon > \varepsilon_{y1}$

$\beta_1 = 8\, (\varepsilon_{y1}/\Delta)^2 + I1 + I2$ for $\varepsilon > \varepsilon_{y2}$

Where:

$I1 = 12(1-E2/E1)\, (\varepsilon_{y2}^2 - \varepsilon_{y1}^2)\, \varepsilon_{y1}/\Delta^3 + 8\, E2/E1\, (\varepsilon_{y2}^3 - \varepsilon_{y1}^3)/\Delta^3$

$I2 = 12/\Delta^3\, [\{(1-E2/E1)\, \varepsilon_{y1} + E2/E1\, \varepsilon_{y2}\}(\Delta^2/4 - \varepsilon_{y2}^2)\,]$

$\beta_2 = [4(\varepsilon_1^3 - \varepsilon_2^3) - 6\, \varepsilon_o\, (\varepsilon_1^2 - \varepsilon_2^2)\,] / \Delta^3$ for $\varepsilon \leq \varepsilon_{y1}$

$\beta_2 = E2/E1\, (1-3\, \varepsilon_o/\Delta) + 3(1-E2/E1)\, (1-4\varepsilon_o/\Delta)(\varepsilon_{y1}/\Delta) + 12(1-E2/E1)\, (\varepsilon_o/\Delta)\, (\varepsilon_{y1}/\Delta)^2 - 4(1-E2/E1)\, (\varepsilon_{y1}/\Delta)^3$ for $\varepsilon_{y2} \geq \varepsilon > \varepsilon_{y1}$

$\beta_2 = I1 + I2 + I3$ for $\varepsilon > \varepsilon_{y2}$

Where:

$I1 = 8\, (\varepsilon_{y1}/\Delta)^3 - 12\, \varepsilon_o/\Delta\, (\varepsilon_{y1}/\Delta)^2$

$I2 = 24/\Delta^3\, [(1-E2/E1)\, \varepsilon_{y1}\, \{(\varepsilon_{y2}^2 - \varepsilon_{y1}^2)/2 - \varepsilon_o\, (\varepsilon_{y2} - \varepsilon_{y1})\} + E2/E1\{(\varepsilon_{y2}^3 - \varepsilon_{y1}^3)/3 - \varepsilon_o/2\, (\varepsilon_{y2}^2 - \varepsilon_{y1}^2)\}\,]$

$I3 = 24/\Delta^3\, [(1-E2/E1)\, \varepsilon_{y1} + E2\, \varepsilon_{y2}]\, [1/2\, \{(\Delta/2)^2 - \varepsilon_{y2}^2)\} - \varepsilon_o\, (\Delta/2 - \varepsilon_{y2}\}]$

The following concrete continuous relationships are considered in addition to the parabolic model in section (D). The numerical integration method is used to determine the values of (β_1 and β_2).

G- Continuous stress-strain relationship [8 & 9] shown in Fig. (4-c):

$\sigma = E\, \varepsilon / [1 + (R-R_E-2)(\varepsilon/\varepsilon_o) - (2R-1)(\varepsilon/\varepsilon_o)^2 + R\, (\varepsilon/\varepsilon_o)^3\,]$

Where:

E = initial modulus of elasticity = σ / ε

E_o = Secant modulus of elasticity = σ_o / ε_o

σ = stress at strain (ε) and σ_o = stress at strain (ε_o)

σ_f = failure stress at failure strain (ε_f)

$R_E = E / E_o$

$R\varepsilon = \varepsilon_f / \varepsilon_o$

$R_F = \sigma_o / \sigma_f$

$R = R_E\, (R_F-1)/ (R\varepsilon - 1)^2 - 1/ R\varepsilon$

H- Uniaxial stress-strain relationship including confinement [10 & 11] shown in Fig. (4-d):

Ascending part: $\sigma_c = \sigma_o\, [1 - (1 - \varepsilon/\varepsilon_o)^A\,]$

Descending part: $\sigma_c = \sigma_o\, e^{-B\, (\varepsilon-\varepsilon o)\, \wedge C}$

Where:

$A = E\, \varepsilon_o/\sigma_o$
$B = (260 + 100/fc')\, e^{-30fcl/fc'}$
$C = 1.2 - 0.006\, fc'$
$f_o = fc' + 4.2\, fcl$
$\varepsilon_o = 0.0007\, (fc')^{1/3} + 0.06\, (fcl/fc')$
$fcl = \rho_s f_{sy}/2\, (1 - \sqrt{s/dc})$
fc' is the compressive strength of the concrete.
ρ_s is the confinement reinforcement index, f_{sy} is the yield strength of the steel, s is the spacing of the confined stirrups and dc is the concrete cover.

I- Continuous model [12] shown in Fig. (4-e):

$\varepsilon = 9 * 10^{-7}\, \sigma + 6 * 10^{-10}\, \sigma^2$

Results and Discussion:

Fig.(5-a) shows the results of inelastic behavior of linear, elasto-plastic, elastic –perfectly plastic, bi-linear and tri-linear stress-strain relationships. As expected the modular ratio (β_1=E'/E) for linear model remain unity for all values of strains, but in elasto-plastic model, the constant value (1) is obtained for strain less than the yielding strain, but is not longer remain constant when the strain exceed this limit, the value of modular ratio is reduced rapidly by the rate ($12\, \varepsilon^3/\Delta^4 - 3\, \varepsilon_y/\Delta^2$). The losses percent in elasticity modulus value are given in the following table:-

Table (1) Losses in Elastic Modulus

$\varepsilon / \varepsilon_y$	β_1	% losses
1	1	0
2	0.625	37.5
3	0.5	50
4	0.375	62.5
5	0.3	70

The rapid losses occurred when the strain exceed the yielding strain.

Because the strain of the elasto-plastic model is not limited, the strain ($3\varepsilon_y$) can be limited as the maximum strain which result (50%) reduction in the value of elasticity modulus. In case of elastic-perfectly plastic model, the value of modular ratio reciprocally reduced by the rate ($-3\, \varepsilon_y/\Delta^2$).

The bending moment capacity coefficient ($\psi = \beta_1\Delta$ = Mh/EI) shown in Fig. (5-b) is linearly increased in linear model as expected and is linear for elasto-plastic model up to yielding strain, then the non-linearity begins beyond this limit and the value of the moment coefficient is gradually increased to the maximum value ($3\varepsilon_y$) which is the solution of the elastic-perfectly plastic model.

In case of bi-linear model [6] and tri-linear [7], the behavior is approximately similar to elasto-plastic model and the rate of decreasing of elasticity modulus value is [$-3(1\text{-}E2/E1)\ \varepsilon_y/\Delta^2 + 12\ (1\text{-}E2/E1)\ \varepsilon_y^3/\Delta^4$], when (E2=0) the same results of elasto-plastic model is obtained. Fig.(6-a) show the effect of modular ratio (E2/E1) of bi-linear model on the inelastic modulus, in all cases the unity value is obtained for strain value up to (ε_y), then sudden drop is occurred with increasing the strain beyond this limit and the slope of the curve increased by amount of (1-E2/E1). At (E2/E1=1) the same results of linear model are obtained and at (E2=0) the results of the elasto-plastic model are obtained. Same behavior is noticed in moment capacity coefficient as shown in Fig. (6-b), the solution is varying between the linear and elasto-plastic results depending on the value of (E2/E1). Slopes of the curves vary between the maximum value at (E2/E1=1) and minimum at (E2/E1=0). Effect of the modular ratio on the inelastic modulus in case of bending plus axial load of the previous two models (bi-linear and tri-linear) are shown in Figs. (11 and 12). This effect can be represented in the following proposed equations:

Bi-linear model: $\beta_1 = 0.71 + 0.29\ E2/E1$ ----------- (18)

$\psi = M\,h / E\,I = 0.0057 + 0.0023\ E2/E1$ ---------- (19)

Tri-linear model: $\beta_1 = 0.47 + 0.53\ E2/E1$ ---------- (20)

$\psi = M\,h / E\,I = 0.0019 + 0.0034\ E2/E1$ ---------- (21)

These equations give results which exactly agreed with the results of elasto-plastic model at (E2/E1=0) and linear model at (E2/E1=1).

Fig.(7-a) shows the plot of inelastic modulus of the concrete continuous models [8 & 9], the linear part is very short up to strain ($0.15\varepsilon_o$), then the value is decreased from unity in linear part to (0.53) of its initial value at peak strain (ε_o) and become (0.34) at final strain (ε_f). The same behavior is noticed in moment capacity coefficient shown in Fig. (7-b). Figs. (8-a &b) show the effect of strain ratio ($R\varepsilon = \varepsilon_f/\varepsilon_o$) on the elasticity modulus and moment capacity of members, as shown the effect is appear when the strain exceed ($2/3\ \varepsilon_o$). The results explain that the increase of final strain with respect to the peak strain cause significant decrease of elasticity modulus and moment capacity. Also the effect of the stress ratio ($R_F = \sigma_o/\sigma_f$) is considered and shown in Figs.(9-a & b), the linear part extend up to strain ($0.15\varepsilon_o$) and the value of elasticity modulus and moment capacity are increased with the increasing of the stress ratio (R_F) up to strain ($0.85\varepsilon_o$), but beyond this limit the behavior is reversed. This point can be termed as an inflection point and at this point (E'/E=0.6 and M=0.001 E I /h), the effects of strain and stress ratios can be expressed in the following proposed equations:

At $\varepsilon = 0.002$ and $R_F=1.5$

$$\beta_1 = 0.25\ (1 + 16/25\ R\varepsilon) \qquad \text{---------} \ (22)$$

$$\psi = M\ h / E\ I = 0.0007\ (1 + 0.31\ R\varepsilon) \qquad \text{---------} \ (23)$$

At $\varepsilon = 0.001$ and $R\varepsilon=1.5$

$$\beta_1 = 0.54\ (1 + 1/3\ R_F) \qquad \text{----------} \ (24)$$

$$\psi = M\ h / E\ I = 0.0053\ (1 + 0.36\ R_F) \qquad \text{----------} \ (25)$$

This behavior explain that increasing the peak stress with respect to final stress has positive effects at low strain values and negative effects at high strain values because the concrete become more brittle and sensitive for strain with increasing of concrete peak stress.

The results obtained using concrete continuous models [5, 10, 11 & 12] are also shown in Figs.(7-a & b) in all cases the linear part is very short within ($0.1\varepsilon_o$ to $0.2\varepsilon_o$) then sudden drop is occurred in the value of the elasticity modulus and lost (50 to 60%) of its initial value in very short strain interval.

In the following discussion, the effect of bending plus axial load is considered for the same previous models. Figs.(10-a & b) show the results using the elasto-plastic model, as shown significant reduction in elasticity modulus value is occurred with increase of strain and applied stress especially when the applied stress exceed ($0.45\ F_y$) and the slope of the curves reduced with the increasing of the stress level and reaches zero slope at high stress level, this mean that the applied stress has the major effect on the elasticity value. The rate of change of the slope is:

$$-3/\Delta^4\ [4(\varepsilon_1^{\,3} - \varepsilon_2^{\,3}) - 6\ \varepsilon_o\ (\varepsilon_1^{\,2} - \varepsilon_2^{\,2})] \qquad \text{for } (\varepsilon \le \varepsilon_y) \quad \text{------} \ (26)$$

and

$$-36/\Delta^4[(\varepsilon_y^{\,3}-\varepsilon_2^{\,3})/3-\varepsilon_o/2(\varepsilon_y^{\,2}-\varepsilon_2^{\,2})+\varepsilon_y\{(\varepsilon_1^{\,2}-\varepsilon_2^{\,2})/2-\varepsilon_o(\varepsilon_1-\varepsilon_2)\}] \quad \text{for } (\varepsilon > \varepsilon_y) \quad \text{-----} \ (27)$$

The effect of the applied stress is more illustrated in Fig.(10-c), the value of the elasticity modulus is exponentially decreased with the applied stress level and the following equation is proposed:

$$E''/E = 0.975\ e^{-1.621\sigma/Fy} \qquad \text{----------} \ (28)$$

In case of using general stress-strain relationship [2], the value of inelastic modulus in gradually reduced up to stress ($0.45\ F_y$), then rapid reduction is occurred beyond this limit, this explain the critical limit that used in working stress design method between (0.4-0.5 F_y). In Elasto-plastic model the reduction (50%) in elasticity modulus value occurred at stress value (0.4 F_y). Also sudden drop is noticed at high strain values, this limit can be termed as a critical limit or failure point due to buckling and the following equation is proposed to estimate the value of the critical limit.

$$\varepsilon_{cr}/\varepsilon_y = 6.425\ (1 - 0.41\ \sigma/Fy) \qquad \text{---------- (29)}$$

Figs.(11-a,b and 12-a,b) show the results using the bi-linear and tri-linear models and the results are approximately same as previous model.

Conclusions:

1- Because the strain of the elasto-plastic model is not limited, the strain ($3\varepsilon_y$) can be limited as the maximum strain which result (50%) reduction in the value of elasticity modulus.
2- In case of elasto-plastic model the behavior is linear up to yielding strain, and then the non-linearity begins beyond this limit.
3- In case of bi-linear model and tri-linear, the behavior is approximately similar to elasto-plastic model, the sudden drop is occurred in the value of elasticity modulus beyond the yielding strain point, the behavior is vary between the linear model and elasto-plastic model .
4- Increasing of strain ratio ($R\varepsilon$) cause significant decrease of elasticity modulus and moment capacity.
5- Elasticity modulus and moment capacity are increased with the increasing of the stress ratio (R_F) up to strain ($0.85\varepsilon_o$), but the behavior is reversed beyond this limit.
6- In case of continuous models, the linear part is very short within ($0.1\varepsilon_o$ to $0.2\varepsilon_o$) then sudden drop is occurred in the value of the elasticity modulus and lost (50 to 60%) of its initial value in very short strain interval.
7- Significant reduction in elasticity modulus value is occurred with increase of applied stress especially when exceeds (0.45 F_y).
8- Suitable equations are proposed to estimate the value of inelastic modulus and moment capacity in term of modular ratio (E2/E1), strain ratio ($R\varepsilon$), stress ratio (R_F) and external applied stress ratio (σ/Fy).

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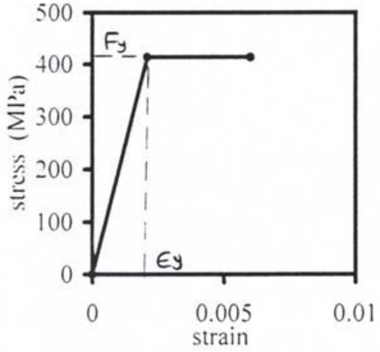
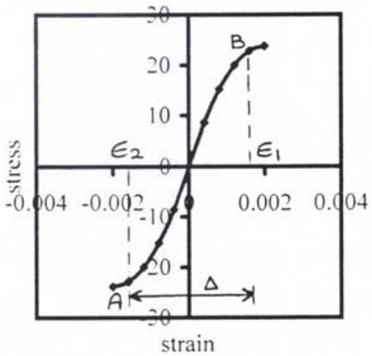


Fig(1):
Idealized stress-strain relationship



(a) General stress-strain relationship

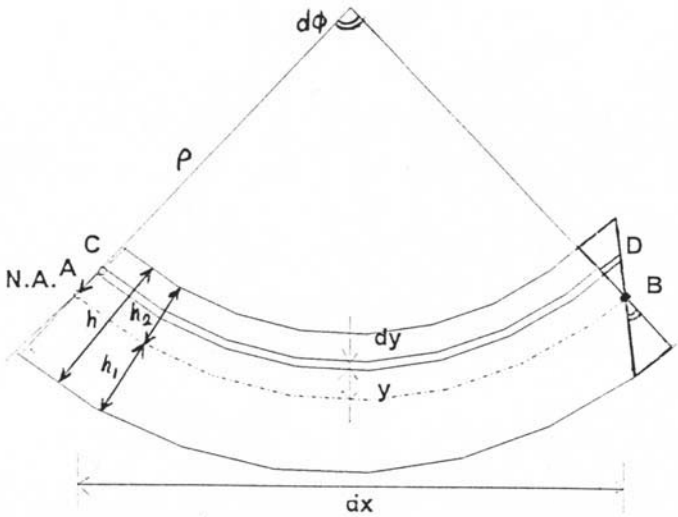
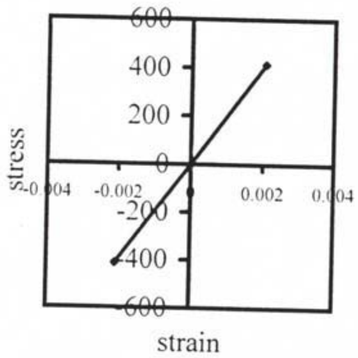
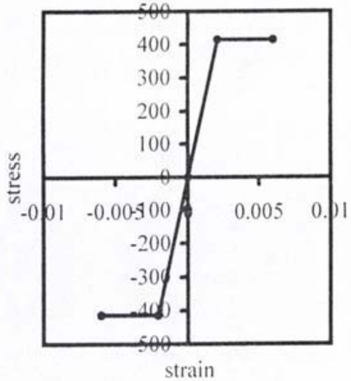


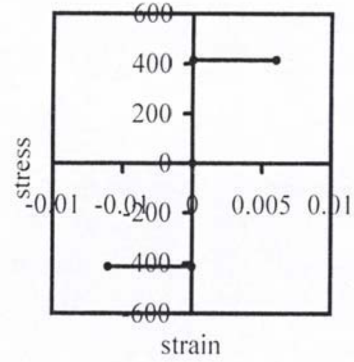
Fig. (2): Bending of Rectangular cross section.



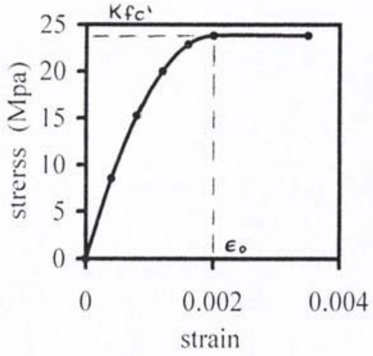
(b) Linear stress-strain relationship



(c) Elasto-Plastic relationship

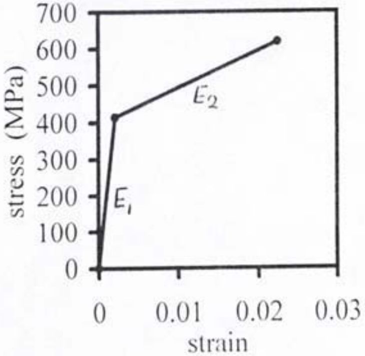


(d) Elastic perfectly plastic relationship

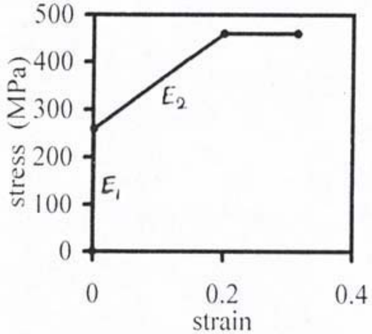


(e) stress-strain relationship Ref.[5]

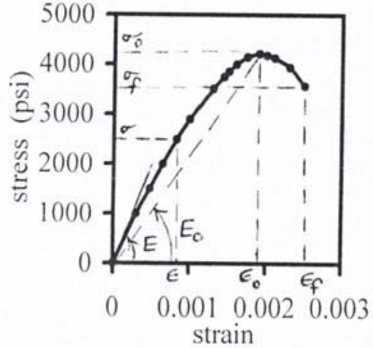
Fig. (3): Different stress – strain relationships



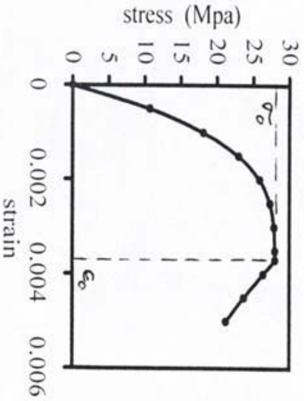
(a) Bi-linear stress – strain Relationship Ref.[6]



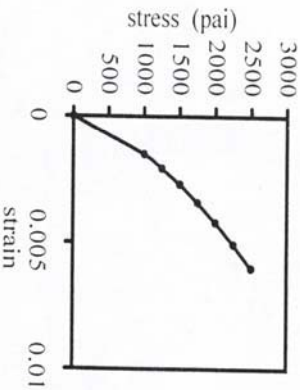
(b) Tri-linear stress – strain Relationship Ref.[7]



(c) Concrete continuous stress – strain relationship Ref.[8 & 9].

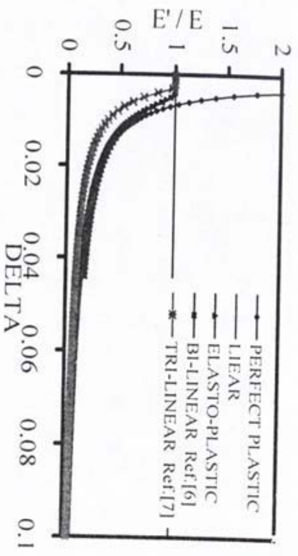


(d) Uniaxial stress-strain relationship including confinement Ref.[10 &11]

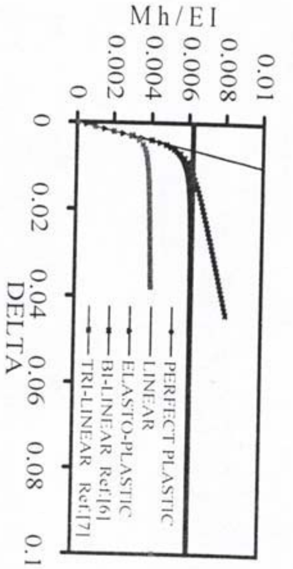


(c) stress-strain relationship [12]

Fig.(4): Different stress – strain relationships for steel and concrete.

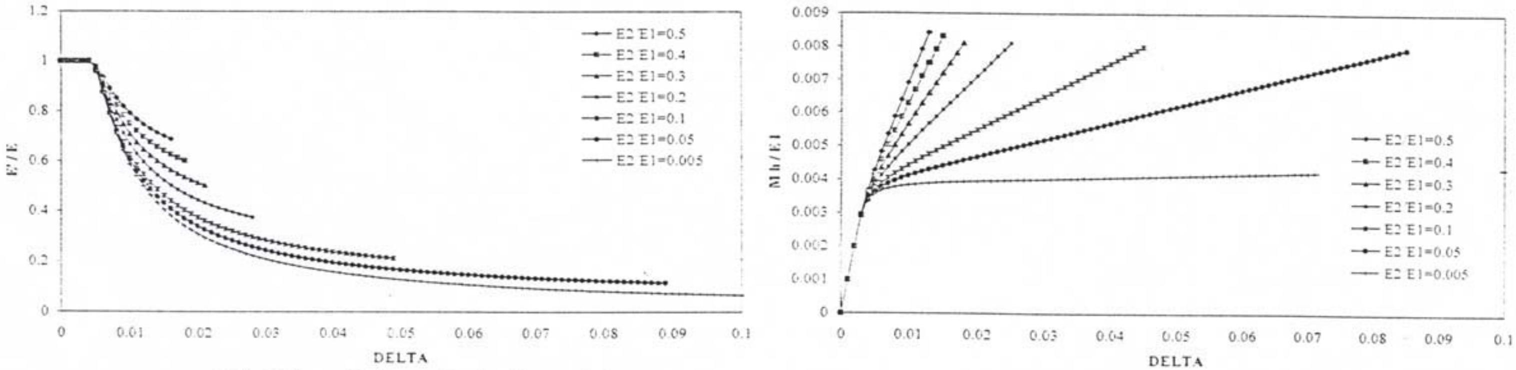


(a) Effect of Delta on inelastic modulus



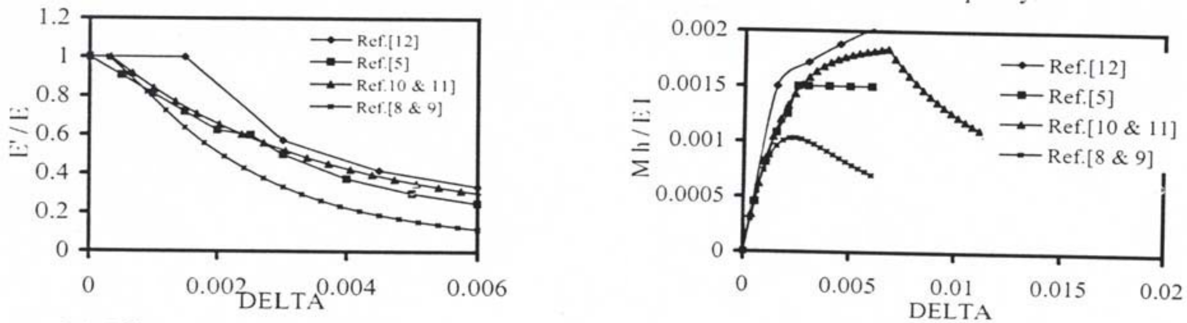
(b) Effect of Delta on moment capacity

Fig. (5): Inelastic modulus and moment capacity in case of pure bending.



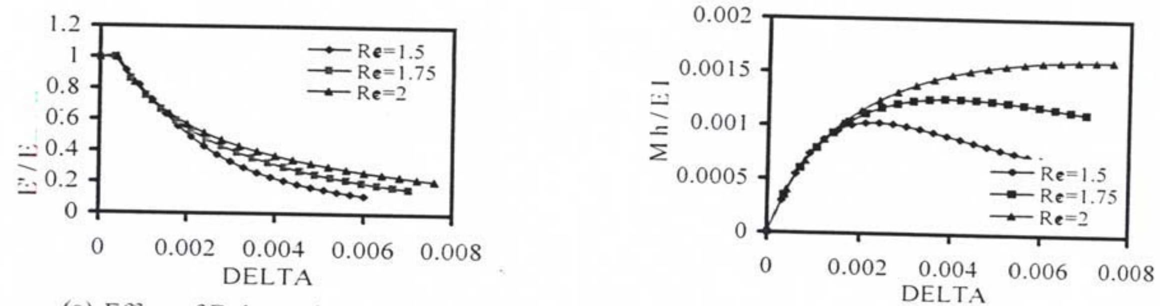
(a) Effect of (Δ) on inelastic modulus. (b) Effect of (Δ) on moment capacity.

Fig. (6): Effect of (Δ) and modular ratio (E2/E1) on inelastic modulus and moment capacity.



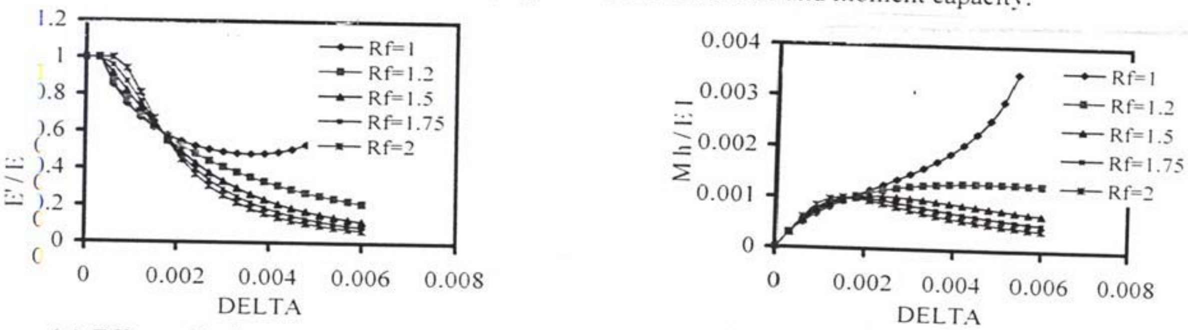
(a) Effect of Delta on inelastic modulus (b) Effect of Delta on moment capacity

Fig. (7): Inelastic modulus and moment capacity in case of pure bending for continuous concrete relationships.



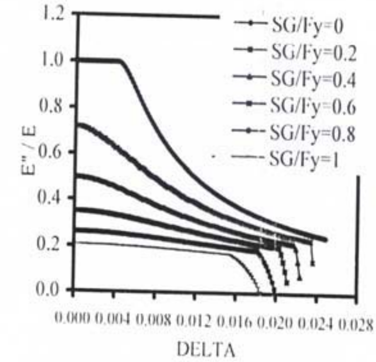
(a) Effect of Delta on inelastic modulus (b) Effect of Delta on moment capacity

Fig. (8): Effect of (Δ) and strain ratio ($R\varepsilon$) on inelastic modulus and moment capacity.

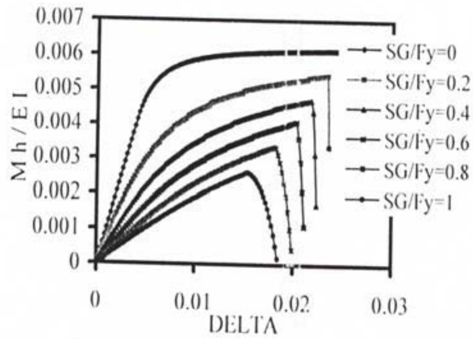


(a) Effect of Delta on inelastic modulus (b) Effect of Delta on moment capacity

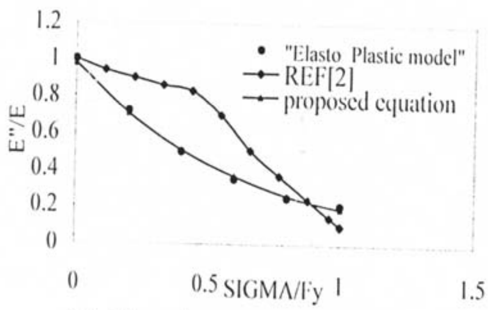
Fig. (9): Effect of (Δ) and stress ratio (R_F) on inelastic modulus and moment capacity.



(a) Effect of Delta on inelastic modulus

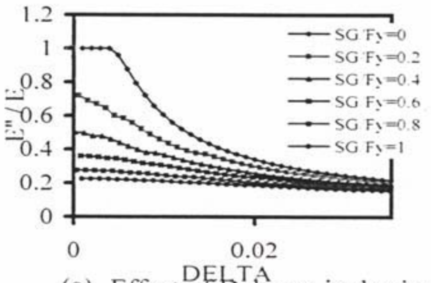


(b) Effect of Delta on moment capacity

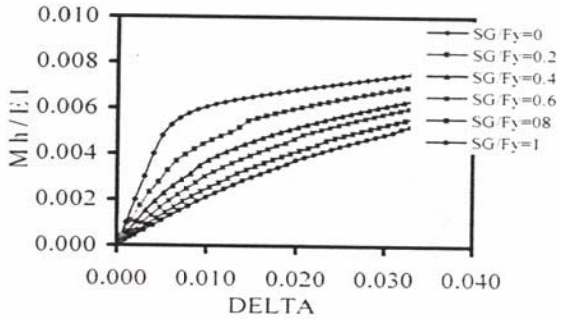


(c) Effect of applied stress ratio on inelastic modulus

Fig. (10): Inelastic modulus and moment capacity in case of bending plus axial stress for elasto – plastic relationship.

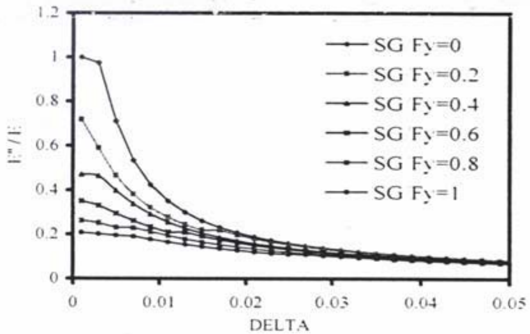


(a) Effect of Delta on inelastic modulus

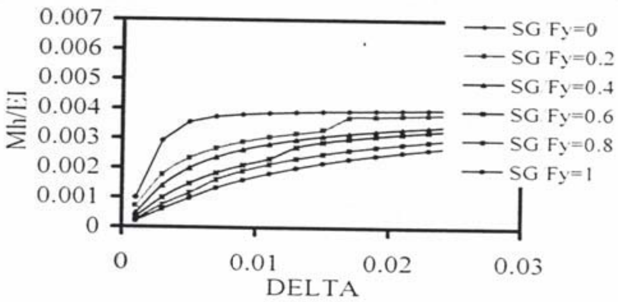


(b) Effect of Delta on moment capacity

Fig. (11): Inelastic modulus and moment capacity in case of bending plus axial stress for bi – linear relationship ref. [6].



(a) Effect of Delta on inelastic modulus



(b) Effect of Delta on moment capacity

Fig. (12): Inelastic modulus and moment capacity in case of bending plus axial stress for tri – linear relationship ref. [7].